

Hot gluon propagator ¹

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Introduction

One of the recently most developing fields of high energy heavy ion physics is the study of the underlying field theory at high temperatures. Despite of the naive picture of the quark gluon plasma as an ideal gas of quarks and gluons stemming from a given interpretation of early lattice gauge theory simulations it is since long clear that the continuum theory is plagued by serious infrared divergences on the supersoft momentum scale $\mathcal{O}(g^2T)$ at the temperature T with a coupling constant g [1, 2]. Those effects, however, cannot be seen on small size lattices.

Even the behavior of the quark gluon plasma on the intermediate, $\mathcal{O}(gT)$ momentum scale is nontrivial, because in this case a thermal field $A \approx T$ contributes with the same order to the covariant derivative (or kinetic momentum) as the pure derivative (momentum): $D = \partial - gA \approx gT - gT$. It means that effects of higher order in the $T = 0$ perturbation theory mixes to effects of lower order in the coupling strength g but higher order in the “gradient expansion” of the thermal background. Therefore a new expansion parameter gT is introduced making possible to resum contributions of high momentum ($\mathcal{O}(T)$) hard loops in the propagators and vertices involving soft ($\mathcal{O}(gT)$) momenta. This method, called “hard thermal loop” expansion, is due to Braaten and Pisarski[3, 4, 6].

As a first application the gluon damping rate, i.e. the imaginary part of the hot gluon self-energy describing loss and gain of gluon numbers in a given momentum bin, has been calculated. This, in contrast to earlier calculations which obtained a gauge dependent result even on the sign of this quantity, became positive and gauge independent for zero momentum.

The HTL resummation describes the electric (Debye) screening, inserting a self energy term into the plasmon (low momentum) propagator, which is obtained by integrating over hard (high momentum) thermal loops. This self-energy is symmetric in color and space-time indices - reminding us to the physical mechanism behind it: charges (currents) are screened by fields induced by the charges (currents) themselves in a linear approximation. Self correlation is by definition symmetric.

Since there are general considerations showing that the zero momentum (static, long wavelength) behavior of the gluon propagator is gauge invariant,

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it has been explicitly calculated so far only in a few gauges (Coulomb gauge, axial gauge). In this article we derive a general form of the HTL gluon propagator including all non-background gauge fixing conditions at once. We show explicitly that this result leads to a gauge invariant interaction energy between covariant currents and the vector potential induced by them.

Gauge invariance of physical observables in phenomena probing the dynamics of elementary particle fields, such as high energy heavy ion collisions are, represents one of the most important guiding principles of modern physics. It emphasizes the importance of *symmetry* in formulating the laws of nature in mathematical terms. To the evolution of this principle in the quantum physics Eugen Wigner (Wigner Jenő) himself contributed a great deal.

Preliminaries

The general form of the gluon propagator and hence the self - energy is obtained by inverting the operator which is the “coefficient” of terms quadratic in vector potential fluctuations by expanding the effective (gauge fixed) action for $S[A + \delta A]$. This way the self-energy describes the inertia of propagating field fluctuations due to their interaction with a background which is made up of the same fields.

By including the underlying symmetry into our description properly we use the covariant derivative operator as an element of the Lie algebra in the hermitic adjoint representation

$$D_\mu = \partial_\mu + iA_\mu$$

with $A_\mu = A_\mu^c T^c$. Here

$$(T^c)^{ab} = -\frac{ig}{\hbar} f^{abc}$$

is the basis of the hermitic adjoint representation of $SU(N)$. The commutator relations of the Lie algebra are

$$[T^a, T^b] = \frac{ig}{\hbar} f^{abc} T^c.$$

In the followings we set $\hbar = 1$ and $g = 1$, at the end all these factors can be properly retained using the above definition of basis vectors.

The field strength tensor is defined by

$$iF_{\mu\nu} = [D_\mu, D_\nu]$$

leading to the familiar components

$$F_{\mu\nu}^c = \partial_\mu A_\nu^c - \partial_\nu A_\mu^c - f^{abc} A_\mu^a A_\nu^b.$$

Noting that in the adjoint representation describing gluons the normalization of the basis elements is

$$\text{tr}(T^a T^b) = N\delta^{ab},$$

the Yang-Mills action can be written as

$$S = \int \frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} = \frac{1}{4N} \int \text{tr}(F_{\mu\nu} F^{\mu\nu}).$$

Equation of motion and inverse propagator can now be derived by inspecting the first and second variation of this action with respect to the vector potential A_μ . Noting that

$$\delta F^{\mu\nu} = [D^\mu, \delta A^\nu] + [\delta A^\mu, D^\nu]$$

we get

$$\delta S = \frac{1}{N} \int \text{tr} ([D^\mu, F_{\mu\nu}] \delta A^\nu) = 0$$

leading to the equation of motion

$$[D^\mu, F_{\mu\nu}] = 0.$$

For the second variation we note that

- i) $\delta^2 S = \frac{1}{N} \int \text{tr} (\delta F_{\mu\nu} \delta F^{\mu\nu} + F_{\mu\nu} \delta^2 F^{\mu\nu})$,
- ii) $\delta^2 F^{\mu\nu} = 2i[\delta A^\mu, \delta A^\nu]$,
- iii) $\int \text{tr} (A[B, C]) = \int \text{tr} (B[C, A]) = \int \text{tr} (C[A, B])$.

Seeking the second variation of the Yang-Mills action in the form

$$\delta^2 S = -\frac{2}{N} \int \text{tr} (\delta A^\mu M_{\mu\nu} \delta A^\nu)$$

we arrive at

$$M_{\mu\nu} = g_{\mu\nu}(D \cdot D) + D_\mu D_\nu - 2D_\nu D_\mu.$$

Here $g_{\mu\nu}$ is the metric tensor of the spacetime, the dot means a scalar product according to this metric, and the covariant derivative operators act like commuting with the D_μ matrix:

$$(D_\mu V)^a = ([D_\mu, V])^a = (\delta^{ac} \partial_\mu - \frac{g}{\hbar} f^{abc} A_\mu^b) V^c.$$

Considering finally gauge fixing conditions of the general form

$$F_\mu A^\mu = 0,$$

where F_μ can be a linear mixture of derivatives (also covariant derivatives with respect to a background field) and heat bath coordinate axis directions, we add the following term to the effective action

$$\frac{\lambda}{2N} \int \text{tr} ([F_\mu, A^\mu][F_\nu, A^\nu]).$$

This gives the usual contribution in terms of color components

$$\frac{\lambda}{2} \int (F_\mu^{ab} A^{b\mu}) (F_\nu^{ac} A^{c\nu}).$$

Varying this part results in the additional term

$$\lambda [[F_\nu, A^\nu], F_\mu]$$

in the equation of motion and simply $\lambda (F_\mu F_\nu)^{ab}$ in the inverse propagator. Note that in some gauges (i.e. background gauges) F_μ is not commutative. We shall specify F_μ later identifying particular gauge fixing prescriptions.

Reference frame

Before presenting the general form of the in-medium hot gluon propagator we introduce some convenient notations. Having a medium (a heat bath) “breaks” the Lorentz invariance. A general treatment is possible transforming the heat bath’s four-velocity n_μ into a general frame.

The orthonormal tetrad (Vierbein)

$$n_\mu = (1, 0, 0, 0) \quad e_\mu = (0, 1, 0, 0)$$

$$f_\mu = (0, 0, 1, 0) \quad g_\mu = (0, 0, 0, 1)$$

which is simple in the medium frame becomes

$$n_\mu = (\cosh\eta, \sinh\eta \cos\theta, \sinh\eta \sin\theta \cos\phi, \sinh\eta \sin\theta \sin\phi)$$

$$e_\mu = \frac{\partial}{\partial\eta} n_\mu \quad f_\mu = \frac{1}{\sinh\eta} \frac{\partial}{\partial\theta} n_\mu \quad g_\mu = \frac{1}{\sinh\eta \sin\theta} \frac{\partial}{\partial\phi} n_\mu$$

characterized by a rapidity η and θ, ϕ angles, in a general Lorentz frame. This dependence is trivial and can be restored at the end of the calculation. Here we work in the heat bath system.

Gluon momentum

The general, off-shell gluon momentum is given by

$$k_\mu = \omega n_\mu + k e_\mu$$

while the longitudinal four vector orthogonal to k_μ is

$$h_\mu = k n_\mu + \omega e_\mu.$$

Here ω is the plasmon energy and k is the absolute value of the three-momentum. We use Minkowski-metric and real time. In this case $h \cdot k = 0$ always and

$$k \cdot k = \omega^2 - k^2 = 0$$

only on-shell. On-shell h_μ and k_μ degenerate to the same light-vector, but the off-shell k_μ is timelike and h_μ is spacelike.

Gauge fixing constraint

The general gauge fixing constraint we deal with is a linear combination of timelike (n_μ) and longitudinal (e_μ) contributions at most linear in the derivative. Let

$$F_\mu = (a + ib\omega)n_\mu + (d + ick)e_\mu$$

be this constraint vector, so the second variation of the effective plasmon action includes the additive term

$$\frac{\lambda}{2} (F_\mu \delta A^\mu)^2.$$

In “classical” gauges $\lambda \rightarrow \infty$ is taken at the end of the calculation. Here we also introduce a vector orthogonal to F_μ and to the two transverse directions f_μ and g_μ ,

$$\tilde{F}_\mu = (d + ick)n_\mu + (a + ib\omega)e_\mu.$$

It is alike F_μ but with the coefficients of n_μ and e_μ interchanged. The orthogonality $F \cdot \tilde{F} = 0$ is easy to see noting that $n \cdot n = 1$, $e \cdot e = -1$ and $n \cdot e = 0$.

Polarization tensor

The polarization tensor is the difference between the equation of motion operator for small-amplitude fluctuations of the elementary field (the vector potential) taken in the presence of a vacuum or medium background and without it. Both the vacuum and medium contributions are orthogonal to the gluon four-momentum k_μ because of leading order current conservation. The general form of the polarization tensor is therefore

$$\Pi_{\mu\nu} = \Pi^L \frac{h_\mu h_\nu}{k \cdot k} + \Pi^T (f_\mu f_\nu + g_\mu g_\nu) - \Pi^{\text{vac}} \left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k \cdot k} \right).$$

Note that $g_{\mu\nu} = n_\mu n_\nu - e_\mu e_\nu - f_\mu f_\nu - g_\mu g_\nu$, and

$$f_\mu f_\nu + g_\mu g_\nu = g_{\mu\nu} - \frac{k_\mu k_\nu}{k \cdot k} + \frac{h_\mu h_\nu}{k \cdot k}$$

is the entirely transverse (physical) projector.

HTL gluon propagator

The resummed HTL gluon propagator is obtained from inverting the equation of motion operator for linearized field fluctuations including the above discussed self-energy insertion. This procedure is equivalent to using the variational derivative of the induced source with respect to the mean field as Blaizot and Iancu have done[5]). It becomes

$$G_{\mu\nu} = -\frac{\tilde{F}_\mu \tilde{F}_\nu}{(F \cdot k)^2 \epsilon^L} - \frac{1}{\lambda} \frac{k_\mu k_\nu}{(F \cdot k)^2} + \left(g_{\mu\nu} - \frac{k_\mu k_\nu - h_\mu h_\nu}{k \cdot k} \right) \frac{1}{k \cdot k - \Pi^{\text{vac}} - \Pi^T}$$

with

$$\epsilon^L = 1 - \frac{1}{(n \cdot k)^2} \Pi^L - \frac{1}{k \cdot k} \Pi^{\text{vac}}.$$

It is orthogonal to the constraint vector F_μ but terms of $\mathcal{O}(1/\lambda)$. In the followings we reconstruct some known particular cases of this propagator.

Covariant gauge, vacuum

In this case $b = c = 1$, $a = d = 0$, $\Pi^L = \Pi^T = 0$. It follows that $F_\mu = ik_\mu$, $\tilde{F}_\mu = ih_\mu$ and $(F \cdot k)^2 = -(k \cdot k)^2$. We get

$$G_{\mu\nu} = \frac{1}{\lambda} \frac{k_\mu k_\nu}{(k \cdot k)^2} + \left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k \cdot k} \right) \frac{1}{k \cdot k - \Pi^{\text{vac}}}.$$

Coulomb gauge, matter

In this case $c = 1$, $a = b = d = 0$, $\Pi^{\text{vac}} = 0$. It follows that $F_\mu = ike_\mu$, $\tilde{F}_\mu = ikn_\mu$ and $(F \cdot k)^2 = -k^4$. We get

$$G_{\mu\nu} = -\frac{n_\mu n_\nu}{k^2 \epsilon^L} + \frac{1}{\lambda} \frac{k_\mu k_\nu}{k^4} - \frac{f_\mu f_\nu + g_\mu g_\nu}{\omega^2 - k^2 \epsilon^T}$$

with

$$\epsilon^L = 1 - \frac{1}{\omega^2} \Pi^L \quad \text{and} \quad \epsilon^T = 1 + \frac{1}{k^2} \Pi^T.$$

Axial gauge, matter

In this case $d = 1$, $a = b = c = 0$, $\Pi^{\text{vac}} = 0$. It follows that $F_\mu = e_\mu$, $\tilde{F}_\mu = n_\mu$ and $(F \cdot k)^2 = k^2$. We get

$$G_{\mu\nu} = -\frac{n_\mu n_\nu}{k^2 \epsilon^L} + \frac{1}{\lambda} \frac{k_\mu k_\nu}{k^2} - \frac{f_\mu f_\nu + g_\mu g_\nu}{\omega^2 - k^2 \epsilon^T}.$$

Temporal gauge, matter

In this case $a = 1$, $b = c = d = 0$, $\Pi^{\text{vac}} = 0$. It follows that $F_\mu = n_\mu$, $\tilde{F}_\mu = e_\mu$ and $(F \cdot k)^2 = \omega^2$. We get

$$G_{\mu\nu} = -\frac{e_\mu e_\nu}{\omega^2 \epsilon^L} - \frac{1}{\lambda} \frac{k_\mu k_\nu}{\omega^2} + \frac{f_\mu f_\nu + g_\mu g_\nu}{\omega^2 - k^2 \epsilon^T}.$$

Coulomb energy

The Coulomb energy is the non-transverse part of the interaction energy $j \cdot A = \frac{1}{2} j \cdot G \cdot j$. Since due to the (leading order) current conservation the four-current j is orthogonal to the (kinetic) four-momentum k it can be decomposed as

$$j^\mu = \frac{\rho}{k} h^\mu + j_1^T f^\mu + j_2^T g^\mu$$

with the color charge density ρ and transverse dynamical current density components j_1^T and j_2^T respectively.

In the kinetic approach[5] an on-shell contribution proportional to $\delta(k \cdot k) k^\mu$ was considered. A conserved current parallel to k^μ can not, however, be off-shell. Only on-shell becomes the vector h^μ degenerate to k^μ as mentioned earlier, so *massless* static charge configurations really represent a four-current in the light-cone direction.

With the above decomposition of the conserved gluonic color current (twice) the interaction energy with the general form of the propagator becomes

$$\int d^4x j_\mu G^{\mu\nu} j_\nu = \int \frac{d^4k}{(2\pi)^4} \frac{\rho(k)\rho(-k)}{k^2 \epsilon^L} + \frac{j_1^T(k)j_1^T(-k) + j_2^T(k)j_2^T(-k)}{\omega^2 - k^2 \epsilon^T}.$$

This result is *gauge invariant*: not only the term depending on the gauge parameter λ drops out from the interaction with a conserved current but because of

$$(h \cdot \tilde{F})^2 = -(k \cdot F)^2$$

the particular form of the gauge fixing constraint F_μ also becomes irrelevant in the final result.

Note that $\rho/k = j^L/\omega$ because of the current conservation therefore there is no need to use a j^L in the parametrization of the four-current. On-shell it is anyway equal to ρ . Problems may occur, however, considering the static limit $\omega \rightarrow 0$. In this case the use of ρ as parameter is physically appropriate, because static modes, which are massive due to Debye screening, induce a time-like causal structure for the induced four-current j_μ . For the nonperturbative chaotic modes, on the other hand, a parametrization with j^L is physical. They namely explore the dynamical behavior ($\omega \neq 0$) of the long wavelength ($k \rightarrow 0$) modes.

Conclusion

In conclusion we derived the general form of the HTL gluon propagator including gauge fixing conditions at most linear in the derivative (kinetic momentum). Using this form we have shown, that the Coulomb energy (interaction energy of static color charges) is independent of the gauge fixing condition. This issue was especially nontrivial in the temporal gauge ($A_0 = 0$).

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References

- [1] A.D.Linde, Rep.Prog.Phys **42**, (1979), 389
- [2] D.J.Gross, R.D.Pisarski, L.G.Yaffe, Rev.Mod.Phys. **53**, (1981), 43
- [3] R.D.Pisarski, Phys.Rev.Lett. **63**, (1989), 1129
- [4] E.Braaten, R.D.Pisarski, Phys.Rev. **D42**, (1990), 2156
- [5] J.-P.Blaizot, E.Iancu, Nucl.Phys. B417, (1994), 608
- [6] E.Braaten, *Solution to the Perturbative Infrared Catastrophe of Hot Gauge Theories*, Preprint NUHEP-TH-94-24, hep-ph/9409434